

Dynamics of spherical metallic particles in cylinder electrostatic separators/purifiers

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Abstract

This paper presents a theoretical analysis of the dynamics of spherical metallic particles in electrostatic separators/purifiers (ESPs). The particle equations of motion are numerically solved in two dimensions using a computational algorithm. The ESPs consist of a pair of conductor cylinder electrodes. The upper cylinder is energized by HVdc, while the lower one is grounded and fixed horizontally on a revolvable axis. Some phenomena and aspects of separation process are explained and depicted including lifting off, impact, “motion collapse” and “sudden bouncing”. The results reveal that the several phenomena depend on initial position, radius and density of the particle, curvature of the cylinder electrodes, distance between the electrodes and amplitude of the applied voltage. Optimization of the parameters is presented in order to get better separation/purification processes.

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1. Introduction

Materials separation and classification applying non-uniform fields between a pair of conducting electrodes was utilized in many industrial applications, e.g. electrostatic separators/sizes [1,2], plate-type separators/purifiers. They have been studied in some published papers [3,4]. In this paper, cylinder-type electrostatic separators/purifiers (ESPs) [5] are introduced to separate binary mixed metals. The ESPs consist of shield device (encloser), motor, collecting boxes, HVdc power, vibrator, filler and a pair of conductor cylinder electrodes etc. The upper cylinder is energized by HVdc, while the lower one is grounded and fixed horizontally on a revolvable axis, and the electrode system and structure could be seen in Fig. 1. Some separation principle and computer program of metal particles' trajectories have been published [5], and a theoretical feasibility analysis of separation and experiments have been carried out. Here some phenomena and aspects are studied including lifting off, impact, “motion collapse”, “sudden bouncing”, trajectory, point of fall and the

factors that control ESP processes. Those phenomena and factors depended on particle's initial position, particle's radius and density, curvature of the cylinder electrodes, gaps between the electrodes and amplitude of the applied voltage. Therefore, the aim of this paper is to present some aspects which were involved in control of ESP processes.

2. Separation principle and forces acting on the particle

It is difficult to carry out a precise separation for a binary mixture of metal particles, just due to their density differences. Separation in vibrated granular matter is coarse and unmanageable [2], so it is difficult to carry out a precise separation using vibration separation. Liquid gravimetric methods could separate a different specific masses mixture, but a separation liquid must be made up, and water pollution will occur. The cylinder-type electrostatic separators/purifiers mentioned in this paper could accomplish a precise separation. The separation structure was shown in Fig. 1. The shield device, collecting boxes and filler are insulated, the upper cylinder electrode (HVE) is energized by HVdc power, while the grounded electrode (GE) is grounded and fixed horizontally on a revolvable axis. The rotation plays a role of vibratory motion and transmission. The particles are

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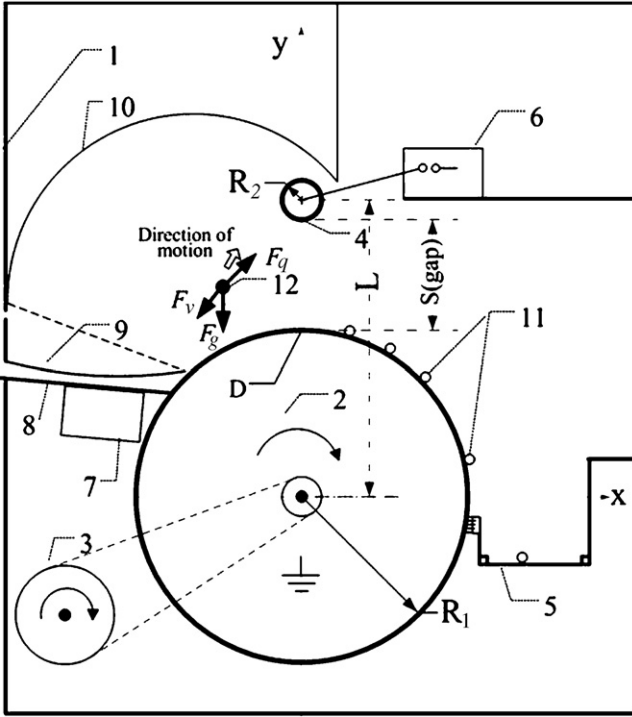


Fig. 1. Electrode system and forces acting on the particle 1, shield device (encloser); 2, GE; 3, motor; 4, HVE; 5, collecting box a (with brush or kick plate); 6, HVdc power; 7, vibrator; 8, filler; 9, collecting box b; 10, back plate; 11, metal particle A and 12, metal particle B.

transferred to the surface of grounded electrode (GE) by the filler and vibrator. Because of the rotation of GE, particles could move as the rotation of GE. For the particles ($\rho_{p1} < \rho_{p2}$, ρ_p represents the density of particles), when HVE is energized by HVdc power, because electric field and the centrifugal force are acted on the particles, the relative lighter particles (ρ_{p1}) could fly from their initial positions easily, but the relative heavier particles (ρ_{p2}) cannot lift off due to $F_q + F_w < F_g$. The relative heavier particles will be still simply immovable relative to surface of GE and move as the rotation of GE. The movement process of particles is fast and instantaneous, and the movement process time of particles is so short that the movement behavior is difficult to be observed and studied.

In order to simulate the particle movement several assumptions are introduced to establish the model and compute the particle trajectory, the assumptions have been discussed in [5]:

- (i) the inelastic particle deformation is not taken into account;
- (ii) the particle charge is assumed to be constant during flying;
- (iii) the coefficient of static friction on the surface of GE is large enough to avoid tumbling of the particle;
- (iv) the distance between two adjacent particles is very big and each particle can be approximated as a material point; the effect among particles is ignored.

A computer program based on the explicit finite method is written to solve the two equations of motion in X and Y directions, and to compute the particle trajectory. Fig. 1 illustrates the forces acting on a flying particle.

The Coulomb force F_q represents the electric force acting on the particle taking charge Q . The particle exists in a two-dimensional electric field E , and the charge is given in formula (1),

$$Q_m = \left(\frac{2\pi^3}{3} \right) \varepsilon_0 r^2 E \quad (1)$$

The force F_q can be decomposed into two direction forces, F_{qx} along the horizontal axis and F_{qy} along the vertical axis. F_{qx} and F_{qy} are expressed in formula (2) [6]:

$$F_{qx} = kQ_m U \left[\frac{y_1}{(a - h_1 + x_1)^2 + y_1^2} - \frac{y_1}{(a + h_1 - x_1)^2 + y_1^2} \right] / \ln \frac{(h_1 + a - R_1)(h_2 + a - R_2)}{(R_1 + a - h_1)(R_2 + a - h_2)} \quad (2.1)$$

$$F_{qy} = kQ_m U \left[\frac{a - h_1 + x_1}{(a - h_1 + x_1)^2 + y_1^2} + \frac{a + h_1 - x_1}{(a + h_1 - x_1)^2 + y_1^2} \right] / \ln \frac{(h_1 + a - R_1)(h_2 + a - R_2)}{(R_1 + a - h_1)(R_2 + a - h_2)} \quad (2.2)$$

where the factor k amounts for image charge, when the particle contacts or impacts an electrode $k=0.832$, otherwise $k=1$ [7]. $h_1 = (L^2 + R_1^2 - R_2^2)/(2L)$, $h_2 = (L^2 - R_1^2 + R_2^2)/(2L)$, $x_1 = \rho \cos(\varphi)$, $y_1 = \rho \sin(\varphi)$, $a = (h_1^2 - R_1^2)^{1/2}$, R_1 represents the radius of grounded cylinder electrode, R_2 is the radius of HVdc cylinder electrode, L is the distance between the electrodes.

The particle experiences a viscous drag force F_d , which opposes the motion. For small particles, the viscous drag can be expressed by Stokes' law, where $F_d = 6\pi r \eta v$, $\eta = 1.83 \times 10^{-5} \text{ kg m}^{-1} \text{ s}^{-1}$ (25°C). The gravitation could be expressed as follows $F_g = 4/3\pi r^3(\rho_p - \rho_s)$, ρ_p is the mass density of particles and ρ_s is the mass density of the atmosphere and its value is 1.29 kg m^{-3} . Lifting electric field E_L can be obtained by the balance of the normal forces (in Y-direction). When the particles are on the surface of GE, a centrifugal force F_w should be considered, it could be expressed as $F_w = mw^2 R_1$, when the particles lift off the surface of GE, $F_w = 0$, w is the palstance of rotation.

3. Computational method

Many parameters influence the particles' motion, for a perfect conducting sphere particle with radius r and a specific mass ρ_p , the particle charges instantly through electrostatic induction, and attain the maximum charge value.

Some particles which can impact the HVE acquire a charge of opposite sign, and then those particles are attracted towards the GE. The effect of the impact between the particle and the HVE was considered in the calculation program. The differential

equations of the particle movement in the air (Fig. 1) are

$$m \frac{d^2x}{dt^2} = F_{qx} + F_{dx} \tag{3}$$

$$m \frac{d^2y}{dt^2} = F_{qy} + F_{gy} + F_{dy}$$

A time-step t_s is defined as a step time, in this paper, $t_s = 0.0001$ s, and this step is so short that error is small enough. For the first point, the initialization values are known. When particles that impact the HVE or GE, then

$$F_{qx_n} = -F_{qx_{n-1}} \tag{4}$$

$$F_{qy_n} = -F_{qy_{n-1}}$$

4. Results and discussion

For all the following figures, the values of the parameters taken for numerical calculation are as given below, except for where it is indicated in the figure and/or figure caption. Particle density $\rho_p = 2700 \text{ kg/m}^3$ for aluminum particles and $\rho_p = 8900 \text{ kg/m}^3$ for copper particles, particle initial position and the applied HVdc 70 kV are given in the figure captions. Subscripts x and y mean the corresponding directions components. Some phenomena and motion trajectory [5] are showed for particles with different radius in Fig. 2.

From Fig. 2, it could be found that particles with different radius lift off at different positions. When the particle lifts off at the point “D” on the surface of GE (Fig. 1), the applied volt-

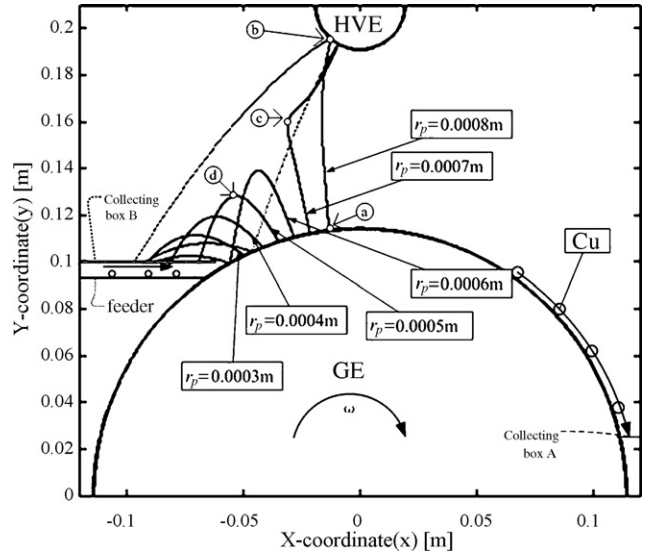


Fig. 2. Particle trajectory in y-x coordinates. The operating parameters were: $0.0003 \text{ m} \leq r_p \leq 0.0008 \text{ m}$, $U_A = 70 \text{ kV}$, $L = 0.21 \text{ m}$, $R_1 = 0.114 \text{ m}$, $R_2 = 0.019 \text{ m}$: a, lifting off; b, impact; c, sudden bouncing and d, motion collapse.

age value is defined to lifting-off voltage. Lifting-off voltage varies with particles of different size or varies with the particles characterized by a different specific mass, it is indicated in Fig. 3(a).

The data represented in Fig. 3(a) show that the lifting-off voltage U_L increases with the increment of the specific mass

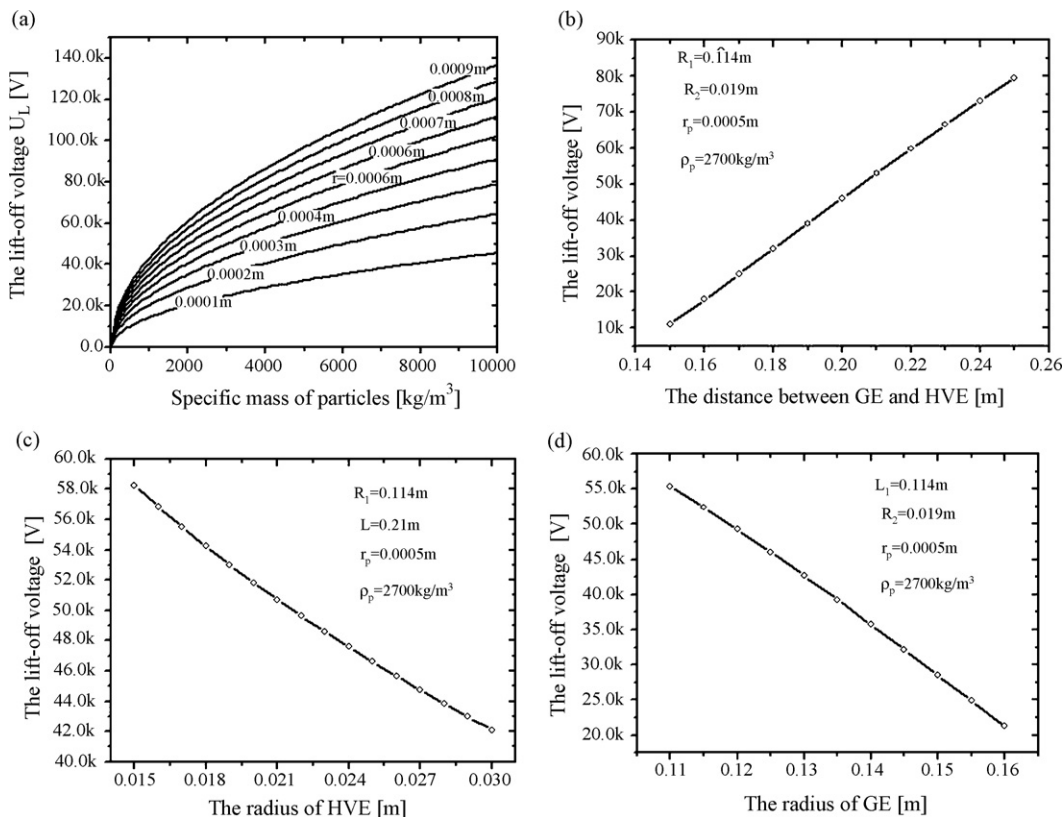


Fig. 3. The Lifting-off voltage U_L for different values of ρ , L , R_1 , R_2 : (a) U_L , specific mass; (b) U_L , L ; (c) U_L , R_1 ; (d) U_L , R_2 .

ρ_p , and U_L increases with the increment of particle radius too. The distance L also influences the U_L , U_L also increases with increment of the distance, it is showed in Fig. 3(b). Curvature of the cylinder electrodes could influence the lifting-off voltage, with increment of the radius of GE and HVE, the lifting-off voltage reduces, which could be displayed in Fig. 3(c) and (d). For this separation structure, a lower lifting-off voltage value is necessary and available, it is easier to control the process, and the energy could be economized under a lower lifting-off voltage. From Fig. 3, three methods could accomplish the object: reducing the distance L between GE and HVE, increasing the radius of HVE, and increasing the radius of GE. But reducing the distance L between GE and HVE is not a better method. It was discovered from experiments that the spark discharge occurred when the distance L was too small. Of course, the other two methods also reduce the distance L between GE and HVE indirectly, but increasing the radius of GE has a greater effect on the distance, while increasing the radius of HVE has almost little effect on the distance L , because the original radius of HVE is very small and a small radius increment of HVE could attain a large descent of U_L , which was showed in Fig. 3(c). So it is a suitable and feasible method to decrease lifting-off voltage by increasing the radius of HVE.

The phenomenon of impact was occurred as shown in Fig. 2. In the process of impact, the direction of motion, vector quantity of velocity and electric force changed, the variation of electric force has been indicated in formula (4). Because the particles and electrodes are metallic, an assumption based on an elastic impact is feasible to some extent, for an elastic impact occurring between the conducting particle and the electrode. The velocity value of the particle after the impact (reflected speed) is equal to that before the impact (incidental speed), and the incidence angle is equal to the reflecting angle. Under such a premise, the impact phenomenon could be explained and expressed by a calculation formula. The velocity vector value before the impact is v_x and v_y , v'_x and v'_y represent the velocity vector value after the impact. The coordinate position where the impact occurs is (X, Y) , the changes of the two angels and velocity are indicated in Fig. 4, and the impact processes can be expressed and explained

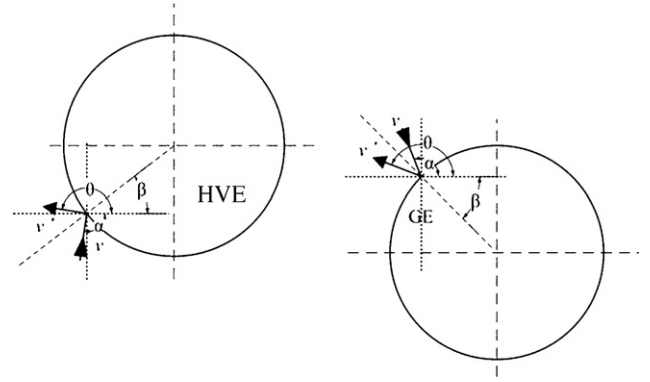


Fig. 4. The process of impact occurs between the conducting particle and the electrode (HVE and GE).

by formula (5) and formula (6),

$$\alpha = \arctan \frac{v_y}{v_x} \tag{5.1}$$

$$\beta = \arctan \frac{Y}{X} \quad (\text{hit GE}) \tag{5.2}$$

$$\beta = \arctan \frac{(Y - L)}{X} \quad (\text{hit HVE})$$

When an impact occurs between the conducting particle and GE, the velocity value after the shock (reflected speed) could be attained by formula (6.1) and formula (6.2),

$$\left\{ \begin{aligned} v'_x &= \sqrt{v_x^2 + v_y^2} \cos(360 - |\alpha| - 2|\beta|) \\ v'_y &= \sqrt{v_x^2 + v_y^2} \sin(360 - |\alpha| - 2|\beta|) \end{aligned} \right\} \quad \alpha > 0 \tag{6.1}$$

$$\left\{ \begin{aligned} v'_x &= \sqrt{v_x^2 + v_y^2} \cos(180 + |\alpha| - 2|\beta|) \\ v'_y &= \sqrt{v_x^2 + v_y^2} \sin(180 + |\alpha| - 2|\beta|) \end{aligned} \right\} \quad \alpha < 0 \tag{6.2}$$

When an impact occurs between the conducting particle and GE, the velocity value after the shock (reflected speed) could be

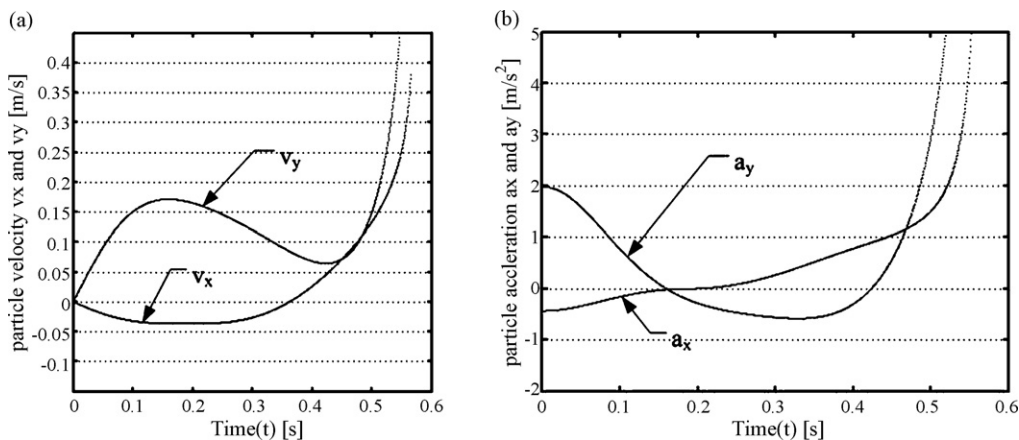


Fig. 5. The variations of (a) the velocity v and (b) the acceleration a with time t for a particle ($r_q = 0.0007$ m).

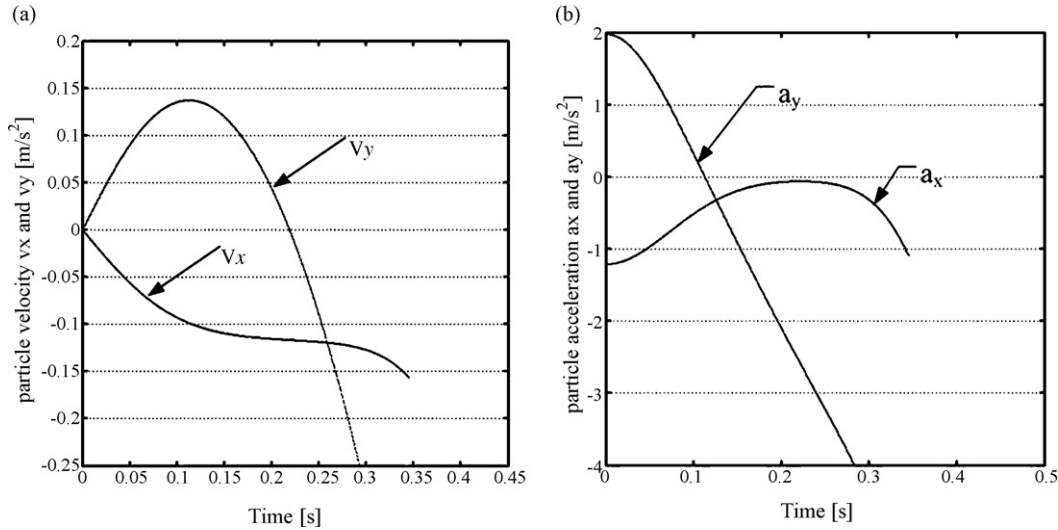


Fig. 6. The variations of (a) the velocity v and (b) the acceleration a with time t for a particle ($r_q = 0.0004$ m).

attained also by formula (6.3) and formula (6.4),

$$\left\{ \begin{array}{l} v'_x = \sqrt{v_x^2 + v_y^2} \cos(180 - |\alpha| + 2|\beta|) \\ v'_y = \sqrt{v_x^2 + v_y^2} \sin(180 - |\alpha| + 2|\beta|) \end{array} \right\} \quad \alpha > 0 \quad (6.3)$$

$$\left\{ \begin{array}{l} v'_x = \sqrt{v_x^2 + v_y^2} \cos(|\alpha| + 2|\beta|) \\ v'_y = \sqrt{v_x^2 + v_y^2} \sin(|\alpha| + 2|\beta|) \end{array} \right\} \quad \alpha < 0 \quad (6.4)$$

The formula (6) expresses the variation of motion, which could depict and explain the process of impact vividly.

Fig. 5 illustrates the variation of both the velocity v and acceleration a with time t for a particle ($r_q = 0.0007$ m). The phenomenon “sudden bouncing” could be explained by Fig. 5. The particle acceleration a_y has a complicated variation including two changes of direction. There is no doubt that the integral electric force F_q should be increscent in the middle process between “lifting off” and “impact”, but electric force of the Y direction F_{qy} is not always increscent. From Fig. 5 when second change of acceleration a_y direction occurs, the particle gets into high electric-field region in Y direction, v_y and v_x increase and a turning point of motion (point “c” in Fig. 2) appears.

Fig. 6 illustrates another phenomenon: the “motion collapse” (point “d” in Fig. 2). The variation of both the velocity v and the acceleration a with time t for a particle ($r_q = 0.0004$ m) is shown in Fig. 6. It indicates clearly that the acceleration in the x direction a_x and the velocity v_x are negative, this means that the particles will move away from the high electric-field location at the X direction. And when the particles lift off the surface of GE, $F_w = 0$ and drag force F_d enhances, so a_y reduces, although F_{ay} enhances. Because of departure at the X direction, at some point, F_{qy} will reduce, and F_{qy} is not enough to overcome resultant force $F_g + F_d$, then a_y changes direction, therefore, the particle is decelerated until its velocity v_y is reversed before hitting the HVE, then the phenomenon “motion collapse” occurred. In this process, the general electric field E or electric force F_q may

increase, but the direction component of electric-field intensity and electric force effected the particles' movement. During the total process, the sign of charge Q is not changed (no impact).

The separation/purification process was analyzed in this paper, the premises of the separation process's success are occurrences of phenomena including lifting off, impact, “motion collapse”, “sudden bouncing”. Therefore, it is necessary to optimize the parameters of the setup to get better separation/purification results.

Analytical results in Fig. 3 show that the curvature of the HVE should be decreased, so that a lower applied voltage just could get same separation/sizing processes effect. From those results, it could be conjectured that the curvature of the GE should be decreased, so that the particle could lift off at a lower departure position and be easier to collect.

A large coefficient of static friction on the surface of GE and rotation of GE are important in a real separation process, the large coefficient of static friction and rotation of GE will play an important role including: (1) driving the metal particles into separation/purification zone; (2) avoiding the metal particles rolling or sliding on the GE surface; (3) generating the centrifugal effect, which has an advantage to make the light metal particles take small lift off from the GE surface. When the metal particles do not contact with the surface of GE, the centrifugal force will no longer exist for those particles. Existing and vanishing of the centrifugal force is one of reasons of “lifting off” and “motion collapse”; (4) driving the large relative density metal particles into the collecting box A. An appropriate coefficient of static friction will lead to a precise separation process.

5. Conclusions

This paper explains and exhibits the separation/purification process. The impact process has been depicted and explained vividly in Eq. (6), and the “motion collapse” and “sudden bouncing” could be explained by curves variation of both the velocity v and the acceleration a with time t for a particle, the direction

component variation of electric-field intensity and electric force is one of prime reasons for influencing the particles' movement.

The lifting-off voltage is an important parameter for separation. And it depends on several factors, such as particle's radius and density, curvature of the cylinder electrodes, and distance between the electrodes. So optimization of the parameters is essential to get better separation/purification processes. It is a suitable and feasible method to decrease lifting-off voltage by increasing the radius of HVE. The curvature of the GE should be decreased also, but the spark discharge should be avoided.

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